Proof that A383500 is the Complement of A382935

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Theorem

Theorem 1. *The sequence A383500 consists exactly of the natural numbers missing from A382935. That is,*

$$\boxed{A383500 = \mathbb{N}^+ \setminus A382935}.$$

Proof

1. Definitions

Definition 1 (Set $\mathcal{A}$ (A383500))**.** *Define*

$$\mathcal{A} = \{n \in \mathbb{N}_0 \mid (i) \ \exists i, \ d_i = 9 \ or \ (ii) \ \forall i, d_i \ odd \Rightarrow d_{i-1} \le d_i\},$$

where $d_1, d_2, \ldots, d_k$ are the decimal digits of n from left to right.

Definition 2 (Set $\mathcal{B}$ (A382935))**.** *Define*

$$\mathcal{B} = \{n \in \mathbb{N}_0 \mid (i') \ \forall i, d_i \ odd \Rightarrow d_{i-1} > d_i\}.$$

2. Disjointness

Lemma 1. *The sets $\mathcal{A}$ and $\mathcal{B}$ are disjoint:*

$$\mathcal{A} \cap \mathcal{B} = \emptyset.$$

Proof. Suppose $n \in \mathcal{A}$.
By definition of $\mathcal{A}$:

- 9 is a digit OR

- Every odd digit is preceded by a smaller digit.

Note there are no digits greater than 9. Thus, n cannot satisfy condition (i') defining $\mathcal{B}$.
Hence, $n \notin \mathcal{B}$.
Similarly, if $n \in \mathcal{B}$, then n violates both of the rules required to be in $\mathcal{A}$.
Thus, $\mathcal{A}$ and $\mathcal{B}$ are disjoint. $\qquad\square$

3. Completeness

Lemma 2. *Every natural number belongs to exactly one of $\mathcal{A}$ or $\mathcal{B}$.*

Proof. Let $n \in \mathbb{N}_0$.

- If n satisfies both condition (i) (every odd digit is followed by a smaller digit) and (ii) (does not end in 1), then $n \in \mathcal{A}$. - Otherwise, if n fails either condition:

- 9 is not a digit of n.

- Any odd digit is preceded by a greater digit, which satisfies (i').

Thus, every positive integer belongs to exactly one of $\mathcal{A}$ or $\mathcal{B}$. □

4. Conclusion

From the disjointness and completeness lemmas, it follows that:

$$\mathcal{A} = \mathbb{N}_0 \setminus \mathcal{B}.$$

Since A383500 enumerates $\mathcal{A}$ and A382935 enumerates $\mathcal{B}$, we conclude:

$$\boxed{\text{A383500} = \mathbb{N}_0 \setminus \text{A382935}}.$$

□

References

[1] OEIS Foundation Inc. (2025), *The On-Line Encyclopedia of Integer Sequences*, published electronically at `https://oeis.org`.
A382935: `https://oeis.org/A382935`
[2] OEIS Foundation Inc. (2025), *The On-Line Encyclopedia of Integer Sequences*, published electronically at `https://oeis.org`.
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