Proof that A383249 is the Complement of A342045

Quinn Savitt

Theorem

Theorem 1. *The sequence A383249 consists exactly of the positive integers missing from A342045. That is,*

$$\boxed{\textit{A383249} = \mathbb{N}^+ \setminus \textit{A342045}}.$$

Proof

1. Definitions

Definition 1 (Set $\mathcal{A}$ (A342045))**.** *Define*

$$\mathcal{A} = \{n \in \mathbb{N}_0 \mid \textit{(i)}\ \forall i,\ d_i \textit{ odd } \Rightarrow d_{i+1} < d_i,\ \textit{ and (ii) } d_k \neq 1\},$$

where $d_1, d_2, \ldots, d_k$ are the decimal digits of n from left to right.

Definition 2 (Set $\mathcal{B}$ (A383249))**.** *Define*

$$\mathcal{B} = \{n \in \mathbb{N}^+ \mid \textit{(i') } \exists i \textit{ such that } d_i \textit{ odd and } d_{i+1} \geq d_i,\ \textit{ or (ii') } d_k = 1\}.$$

That is, $\mathcal{B}$ contains all positive integers that either contain an odd digit immediately followed by a digit greater than or equal to it, or end in 1.

2. Disjointness

Lemma 1. *The sets $\mathcal{A}$ and $\mathcal{B}$ are disjoint:*

$$\mathcal{A} \cap \mathcal{B} = \emptyset.$$

Proof. Suppose $n \in \mathcal{A}$.
By definition of $\mathcal{A}$:

- Every odd digit is immediately followed by a smaller digit.
- The last digit is not 1.

Thus, n cannot satisfy condition (i') or (ii') defining $\mathcal{B}$.
Hence, $n \notin \mathcal{B}$.
Similarly, if $n \in \mathcal{B}$, then n violates one of the rules required to be in $\mathcal{A}$.
Thus, $\mathcal{A}$ and $\mathcal{B}$ are disjoint. □

3. Completeness

Lemma 2. *Every positive integer belongs to exactly one of $\mathcal{A}$ or $\mathcal{B}$.*

Proof. Let $n \in \mathbb{N}^+$.

- If n satisfies both condition (i) (every odd digit is followed by a smaller digit) and (ii) (does not end in 1), then $n \in \mathcal{A}$. - Otherwise, if n fails either condition:

- If an odd digit is followed by a digit greater than or equal to it, then $n \in \mathcal{B}$ by (i').

- If the last digit is 1, then $n \in \mathcal{B}$ by (ii').

Thus, every positive integer belongs to exactly one of $\mathcal{A}$ or $\mathcal{B}$. □

4. Conclusion

From the disjointness and completeness lemmas, it follows that:

$$\mathcal{B} = \mathbb{N}^+ \setminus \mathcal{A}.$$

Since A342045 enumerates $\mathcal{A}$ and A383249 enumerates $\mathcal{B}$, we conclude:

$$\boxed{\text{A383249} = \mathbb{N}^+ \setminus \text{A342045}}.$$

□

References

[1] OEIS Foundation Inc. (2025), *The On-Line Encyclopedia of Integer Sequences*, published electronically at `https://oeis.org`.
A342045: `https://oeis.org/A342045`